C3 TRIGONOMETRY Worksheet C

1 Use the identity $\sin^2 x + \cos^2 x \equiv 1$ to obtain the identities

a $1 + \tan^2 x \equiv \sec^2 x$ **b** $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

2 **a** Given that $\tan A = \frac{1}{3}$, find the exact value of $\sec^2 A$.

b Given that $\operatorname{cosec} B = 1 + \sqrt{3}$, find the exact value of $\cot^2 B$.

c Given that $\sec C = \frac{3}{2}$, find the possible values of $\tan C$, giving your answers in the form $k\sqrt{5}$.

3 Solve each equation for θ in the interval $0 \le \theta \le 2\pi$ giving your answers in terms of π.

a $3\sec^2\theta = 4\tan^2\theta$ **b** $\tan^2\theta - 2\sec\theta + 1 = 0$

c $\cot^2\theta - 3\operatorname{cosec}\theta + 3 = 0$ **d** $\operatorname{cosec}^2\theta + \cot^2\theta = 3$

e $\sec^2\theta + 2\tan\theta = 0$ **f** $\operatorname{cosec}^2\theta - \sqrt{3}\cot\theta - 1 = 0$

4 Solve each equation for x in the interval $-180° \le x \le 180°$.
Give your answers to 1 decimal place where appropriate.

a $\tan^2 x - 2\sec x - 2 = 0$ **b** $2\operatorname{cosec}^2 x + 2 = 9\cot x$

c $\operatorname{cosec}^2 x + 5\operatorname{cosec} x + 2\cot^2 x = 0$ **d** $3\tan^2 x - 3\tan x + \sec^2 x = 2$

e $\tan^2 x + 4\sec x - 2 = 0$ **f** $2\cot^2 x + 3\operatorname{cosec}^2 x = 4\cot x + 3$

5 Solve each equation for x in the interval $0 \le x \le 360°$.

a $\cot^2 2x + \operatorname{cosec} 2x - 1 = 0$ **b** $8\sin^2 x + \sec x = 8$

c $3\operatorname{cosec}^2 x - 4\sin^2 x = 1$ **d** $9\sec^2 x - 8 = \operatorname{cosec}^2 x$

6 Prove each of the following identities.

a $\operatorname{cosec}^2 x - \sec^2 x \equiv \cot^2 x - \tan^2 x$ **b** $(\cot x - 1)^2 \equiv \operatorname{cosec}^2 x - 2\cot x$

c $(\cos x - 2\sec x)^2 \equiv \cos^2 x + 4\tan^2 x$ **d** $\sec^2 x - \sin^2 x \equiv \tan^2 x + \cos^2 x$

e $(\tan x + \cot x)^2 \equiv \sec^2 x + \operatorname{cosec}^2 x$ **f** $(\sin x - \sec x)^2 \equiv \sin^2 x + (\tan x - 1)^2$

g $\sec^2 x + \operatorname{cosec}^2 x \equiv \sec^2 x \operatorname{cosec}^2 x$ **h** $\sec^4 x + \tan^4 x \equiv 2\sec^2 x\tan^2 x + 1$

7 Prove that there are no real values of x for which

$$4\sec^2 x - \sec x + 2\tan^2 x = 0.$$

8 **a** Prove the identity

$$\operatorname{cosec} x \sec x - \cot x \equiv \tan x.$$

b Hence, or otherwise, find the values of x in the interval $0 \le x \le 360°$ for which

$$\operatorname{cosec} x \sec x = 3 + \cot x,$$

giving your answers to 1 decimal place.